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Journal of Hydrology 238 (2000) 65–77

Journal of Hydrology

www.elsevier.com/locate/jhydrol

Steady water flow around parabolic cavities and through parabolic inclusions in unsaturated and saturated soils

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Received 13 December 1999; revised 21 July 2000; accepted 21 August 2000

Abstract

In this paper, we undertake the study of two closely related groundwater flow problems, both two-dimensional, steady and Darcian, and moreover involving parabolic inclusions. First we consider unsaturated flow for which the conductivity depends exponentially on the pressure. Second, we consider saturated flow and an inclusion with a differing, constant, conductivity from the exterior material. We apply the method of separation of variables, conformal mappings, and the Schwarz reflection principle. The distributions of the Kirchhoff potential, specific discharge, and flow net are derived in an explicit analytic form. We show the focusing/diverting properties of a parabola with a more/less permeable interior than the ambient medium, the location of the hinge point and separatrice, and the Maxwell–Philip uniformity of the flow in the interior zone.

Keywords: Infiltration; Refraction; Capillarity; Hermite polynomials; Complex analysis

1. Introduction

Analytical solutions for steady two-dimensional unsaturated flows are based on stringent simplifications of the matrix–fluid properties. Soil is usually assumed to be homogeneous and flow is postulated to be Darcian with a specified dependence of the hydraulic conductivity k (LT^{-1}) on the soil–water potential Ψ (L). For an exponential $k-\Psi$ function the catalogue of explicit solutions is presented in the reviews of Pullan (1990) and Clothier et al. (1995). These solutions serve as benchmarks for numerical and experimental studies of infiltration-induced flows, which are disturbed by natural (cavities, faults, anticlines, etc.) or artificial (capillary barriers) boundaries diverting the vertical route of water (Wang et al., 1999; Warrick et al., 1996).

To our knowledge, all 2D and 3D analytical solutions in vadose zone hydrology are based either on the Dirichlet (given pressure or moisture content) or Neuman (given flux) boundary conditions. However, most soils and rocks include macroheterogeneities (lenses, wedges, diapirs) such that refraction occurs on the interfaces and one has to consider simultaneously flow in two adjacent zones of contrasting hydraulic properties. Experimental visualization of refraction is given by Lehr and Wright (1963) for saturated flows and by Stephens (1996) for unsaturated conditions. Flow is either impeded or accelerated by matrix heterogeneities and streamlines are essentially curvilinear.

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0022-1694/00/$ - see front matter
PII: S0022-1694(00)00310-3